How Machines Explore, Conjecture, and Discover Mathematics
-= Agentic AI in Mathematics =-

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Rundgespräch Angewandte Mathematik

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ZUSE INSTITUTE BERLIN

TU berlin

Berlin Mathematics Research Center
MATH+

What is this talk about?

Introduction

A personal, highly-biased, and incomplete take of what AI can do in Mathematics et al.

Why? AI methods increasingly shape how scientific discovery is performed, configured, and accelerated.

Today: Two (three-ish) recent use-cases / perspectives from my own work.

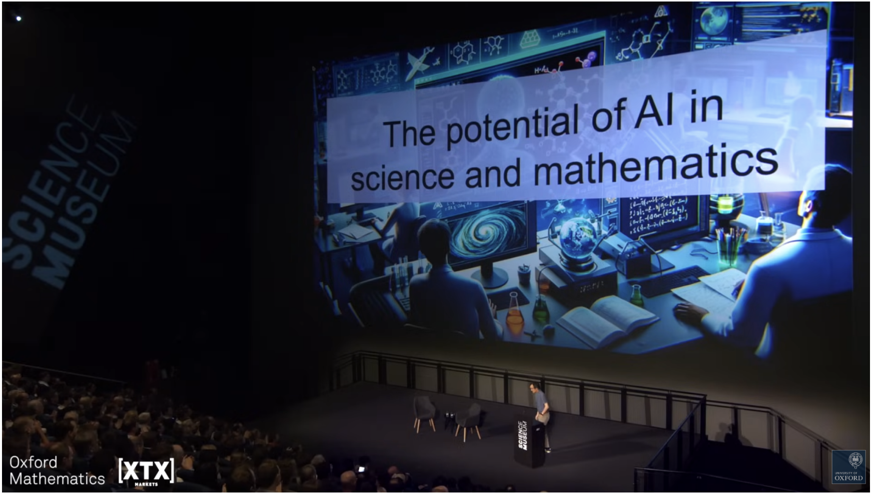
Outline

- A bit of high-level perspective
- Agentic AI Researcher framework
- Neural Colorings of the plane

(Hyperlinked) References are not exhaustive; check references contained therein.

What is this talk about?

AI and Mathematics et al



[The Potential for AI in Science and Mathematics - Terence Tao]

What is this talk about?

AI and Mathematics et al

Various levels of co-creation. [Haase and Pokutta, 2026]

- Digital Pen: basically like autocorrect, bibtex lookup, etc. "2000s"
- AI Task Specialist: ChatGPT, Claude, Gemini, etc. 2022 - 2025
- AI Assistant: Agents with integrated tools, verification, etc. 2025 -
- AI Co-creator: Fully integrated, autonomous, co-creator 2027(??) -

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Capabilities are impressive but unstable:

- SOTA models achieve post-PhD level scores on benchmarks, yet in day-to-day use make trivial (logical) errors.
- No hard verification of results and randomness across runs.
- Prompting and scaffolding are still a challenge.
- Availability of tools for verification etc crucial.

What is this talk about?

Mathematics with computers is not new

Various high-profile examples from the past.

- Four Color Theorem: massive computer-based case checking

  [Appel and Haken, 1977, Robertson et al., 1997]

- Kepler Conjecture / Hales' Theorem: extensive computer verification

  [Hales et al., 2017]

- Classification of Finite Simple Groups: Formal verification with Lean/Coq
- Boolean Pythagorean Triples Problem: A spectacular 200TB SAT-solver proof.

  [Heule et al., 2016]

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Crucial role in computational mathematics / scientific computing

- Finite Elements
- Numerical Simulations
- Optimization
- Engineering
- ...

The Agentic Researcher

A Practical Guide to AI-Assisted Research
in Mathematics and Machine Learning

joint work with: Max Zimmer, Nico Pelleriti,
Christophe Roux

preprint (2026)
https://arxiv.org/abs/2603.15914

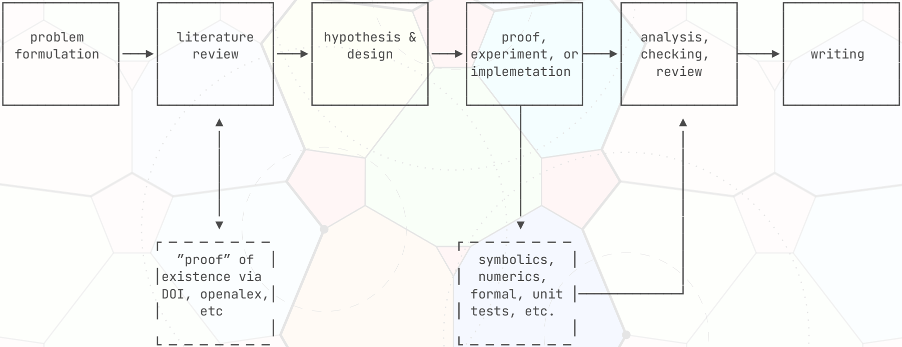
Partially supported by ExC MATH+ Project EF-LiOpt-3
Agent AI in Mathematics

[Zimmer et al., 2026]

AI in the Scientific Discovery Workflow

The agentic researcher framework

AI systems can be used at almost any stage of the scientific workflow.

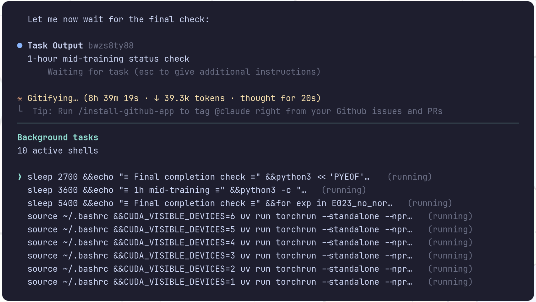


(workflow simplified. for illustration only.)

Note. The researcher needs to be in charge and responsible(!!) throughout.

Building a practical tool

The agentic researcher framework



Screenshot from a run of the framework.

- Born out of the MATH+ project *Agentic AI in Mathematics*.
- Refined over roughly 1.5 years of day-to-day research use.
- Sandboxed CLI agents with project instructions, tools, Git, LaTeX, and GPU-backed experiments.
- Puts a premium on verification: record everything, verify citations, and verify before claiming.

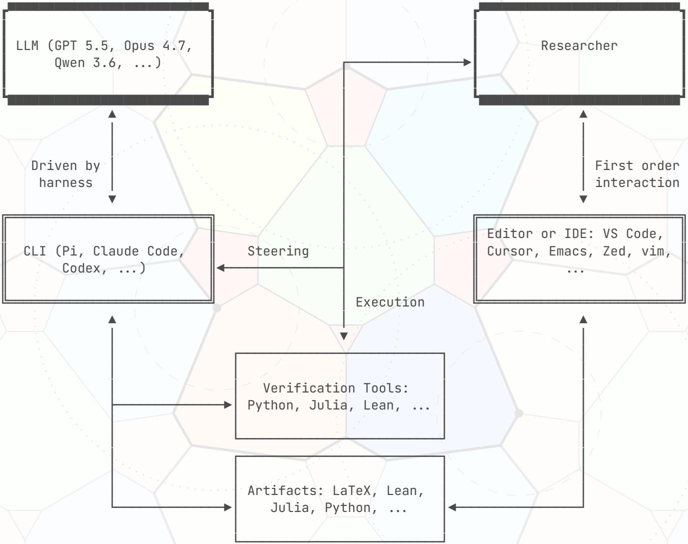
[blog post] [arXiv] [GitHub]

Does it works? Several real-world use-cases in paper

1. Convergence lower bounds for Frank-Wolfe on uniformly convex sets
2. Multi-Variable Dual Tightening for Boscia.jl for MINLPs
3. Weight Reconstruction in LLM pruning
4. ...

Interacting with the System

The agentic researcher framework



Frank-Wolfe Beyond 1/t Convergence

preprint
https://arxiv.org/abs/2604.28006

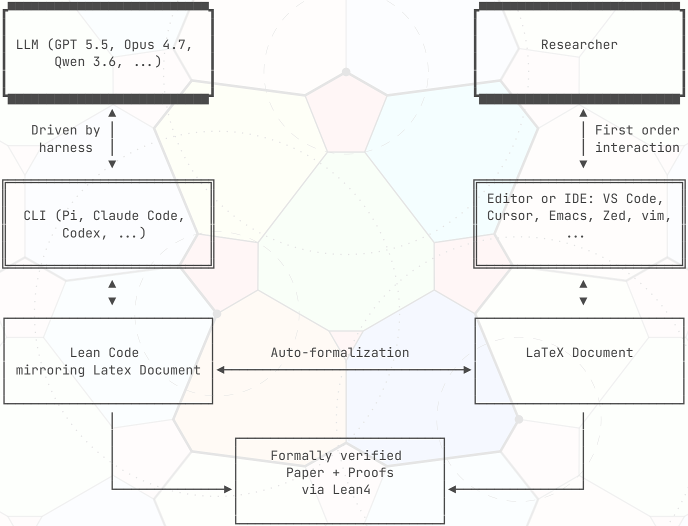
Partially supported by ExC MATH+ Project EF-LiOpt-3

Agent AI in Mathematics

[Pokutta, 2026]

End-2-End Formal Verification with Lean 4

The agentic researcher framework



When generation is cheap, verification is everything.

[Pokutta, 2026]

The Hadwiger-Nelson Problem

joint work with: Aldo Kiem, Konrad Mundinger,
Christoph Spiegel, Max Zimmer

ICML 2025 (oral)
https://arxiv.org/abs/2404.05509

Partially supported by ExC MATH+ Project EF-LiOpt-3

Agent AI in Mathematics

[Mundinger et al., 2025]

The Hadwiger-Nelson Problem

Problem (Nelson 1950, also: Gardner, Moser, Erdős, Harary, Tutte, ...)

What is the smallest number of colors sufficient for coloring the plane in such a way that no two points of the same color are at a unit distance apart?

Infinite graph with vertex set $\mathbb{E}^2$ and edges $\{x, y\}$ for any $x, y \in \mathbb{E}^2$ with $\|x - y\| = 1$

$\Rightarrow$ chromatic number of the plane $\chi(\mathbb{E}^2)$

Theorem

Assuming Axiom of Choice (AoC): [Bruijn and Erdos, 1951]

Any graph is k-colorable iff every finite subgraph of it is k-colorable.

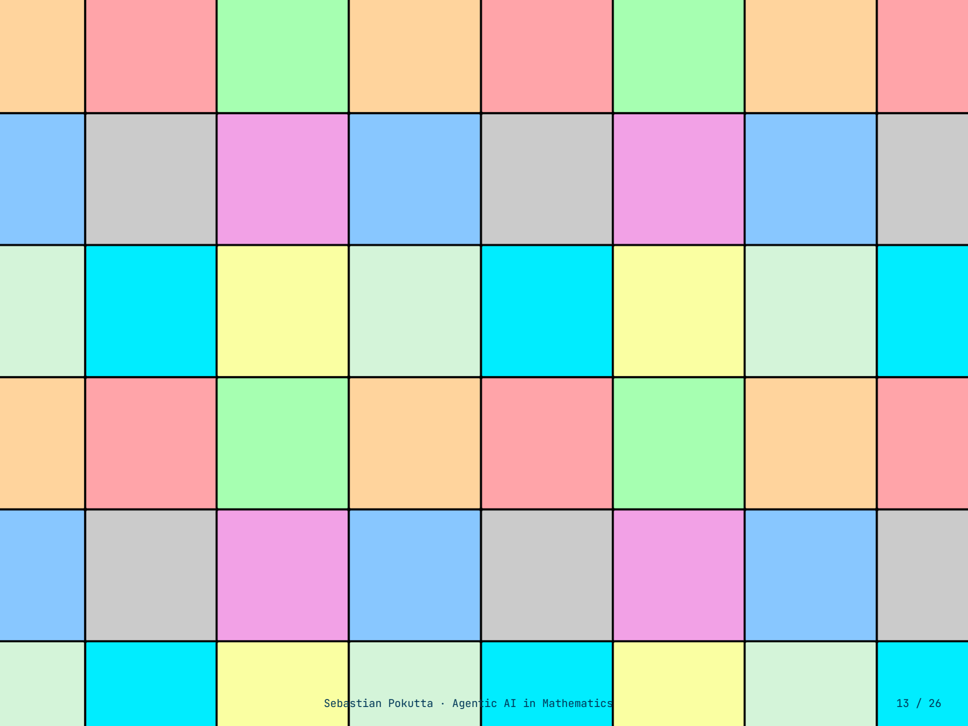
This problem has a long and complicated history... over 14 pages in [Soifer, 2024]

Upper bounds through colorings

The Hadwiger-Nelson Problem

Explicit colorings $g : \mathbb{E}^2 \to [c] := \{1, \ldots, c\}$, usually derived through tesselations using simple polytopal shapes, give

$$5 \leq \chi(\mathbb{E}^2) \leq \ldots$$



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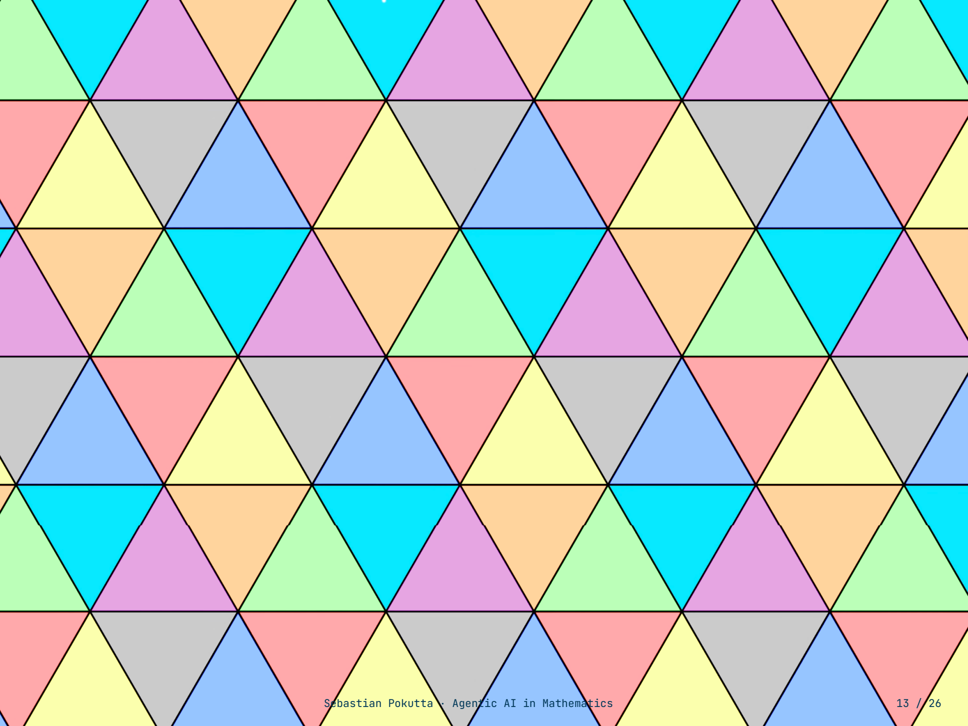

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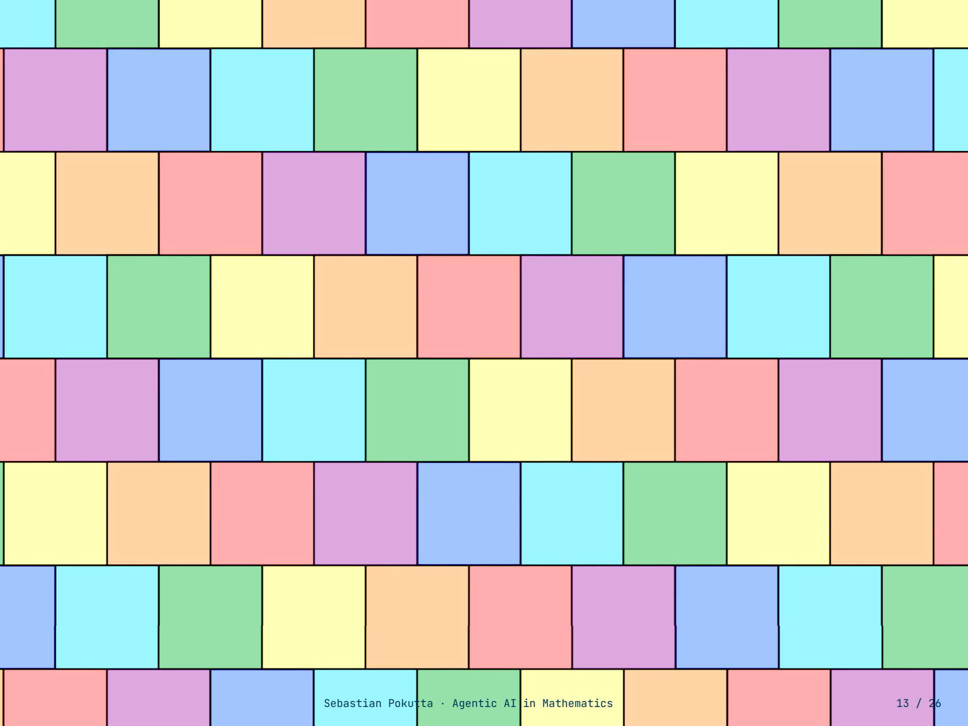
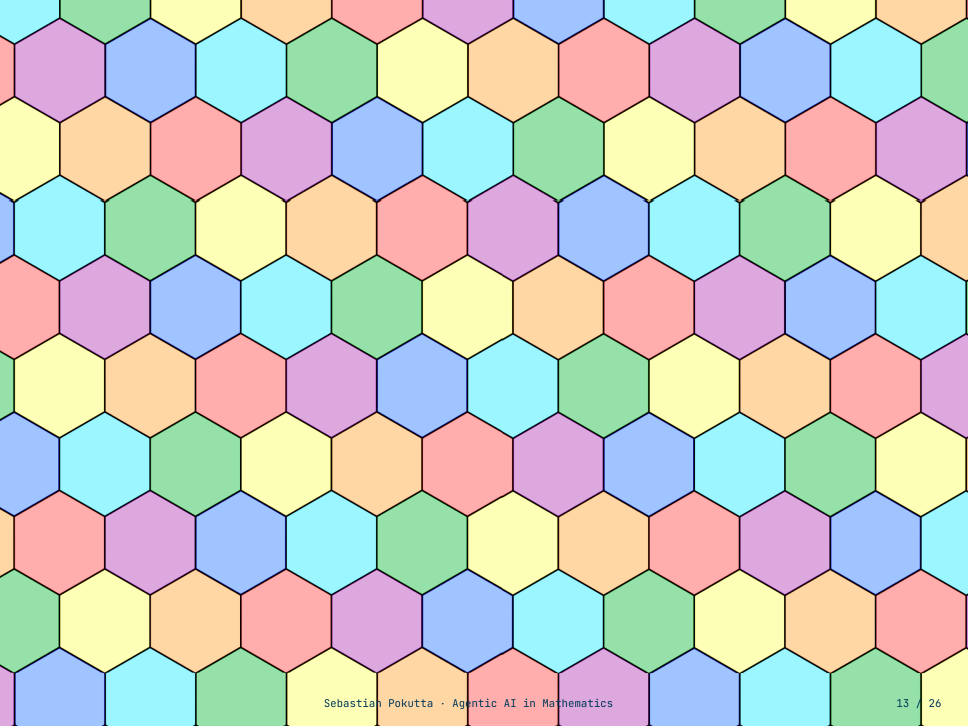
$$5 \leq \chi(\mathbb{E}^2) \leq 8.$$





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Explicit colorings $g : \mathbb{E}^2 \to [c] := \{1, \ldots, c\}$, usually derived through tesselations using simple polytopal shapes, give

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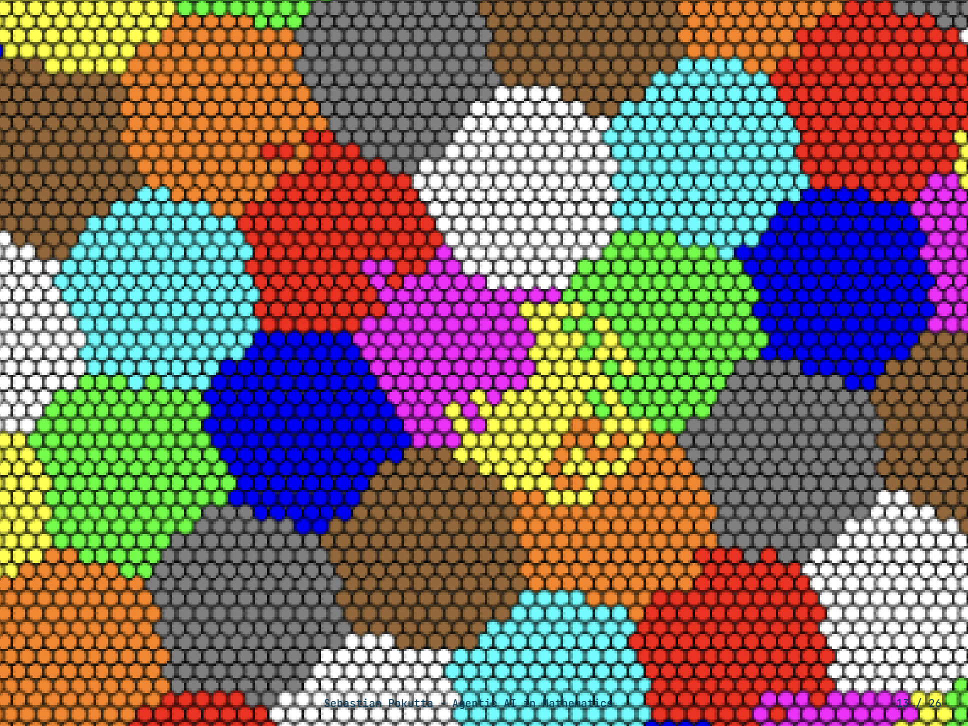
Question. Can we use computers to find admissible colorings $g : \mathbb{E}^2 \to [c]$, i.e.,

$$\{x \in \mathbb{E}^2 \mid \exists y \in B_1(x) : g(x) = g(y)\} = \emptyset?$$

... attempts, e.g., via discretization and SAT solvers...



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Idea. Use a parameterized and easily differentiable family $g_\theta : \mathbb{E}^2 \to \Delta_c$ and find

$$\arg\min_\theta \mathbb{E}\left[\int_{B_1(x)} g_\theta(x) \cdot g_\theta(y)\, dy \,\middle|\, x \in \mathbb{E}^2\right].$$

Key Point. Approach is continuous in nature.

New upper bounds via machine learning?

Can we improve the upper bound?

Neural Networks as Colorings

Idea. Use gradient descent to train a feedforward network g_θ to minimize

$$L(\theta) = \int_{[-b,b]\times[-b,b]} \int_{B_1(x)} g_\theta(x) \cdot g_\theta(y)\, dy\, dx$$

for some reasonable $b \in \mathbb{R}$?

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$$\nabla_\theta L(\theta) \approx \hat{\nabla}_\theta L(\theta) \doteq \frac{1}{m} \sum_{i=1}^{m} \nabla_\theta \, g_\theta(x^{(i)}) \cdot g_\theta(y^{(i)}),$$

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$$\theta_{k+1} = \theta_k - \alpha_k \hat{\nabla}_\theta L(\theta).$$

$\Rightarrow$ Very flexible approach "Deep Annealing"
(also: tropicalization of loss function aka softmax... "minimize the max")

Unfortunately this coloring was already known...

Neural Networks as Colorings

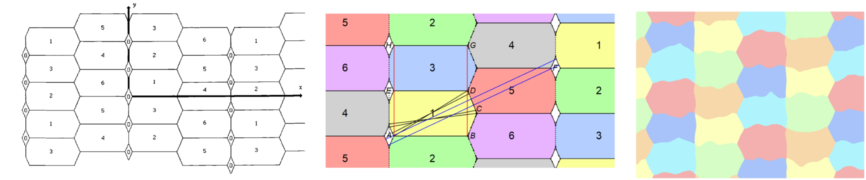

FIG. 3. A good 7-coloring of ($\mathbb{R}^2$, 1).

Theorem

99.985% of the plane can be colored with 6 colors such that no two points of the same color are a unit distance apart. [Pritikin, 1998, Parts, 2020]

Corollary

Any unit distance graph with chromatic number 7 must have at least $6\,992$ vertices.

$\Rightarrow$ While coloring was known already maybe on the right track?

Off-diagonal variant

Going off-diagonal

Neural Networks as Colorings

If we cannot solve the original problem, we study variants of it:

Going off-diagonal

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A c-coloring of the plane has **type** $(d_1, \ldots, d_c)$
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Problem (Soifer in Nash and Rassias' *Open Problems in Mathematics*)

Determine the continuum of six-colorings

$$X_6 = \{d \mid (1,1,1,1,1,d) \text{ can be realized}\}.$$

[Soifer, 1994a, Nash and Rassias, 2016]

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Status. Six-colorings exist for:

1. $d = 1/\sqrt{5}$ [Soifer, 1992]
2. $d = \sqrt{2} - 1$ [Hoffman and Soifer, 1993, 1996]
3. Family with $0.414 \approx \sqrt{2} - 1 \leq d \leq 1/\sqrt{5} \approx 0.447$ [Hoffman and Soifer, 1996, Soifer, 1994b, 2009]

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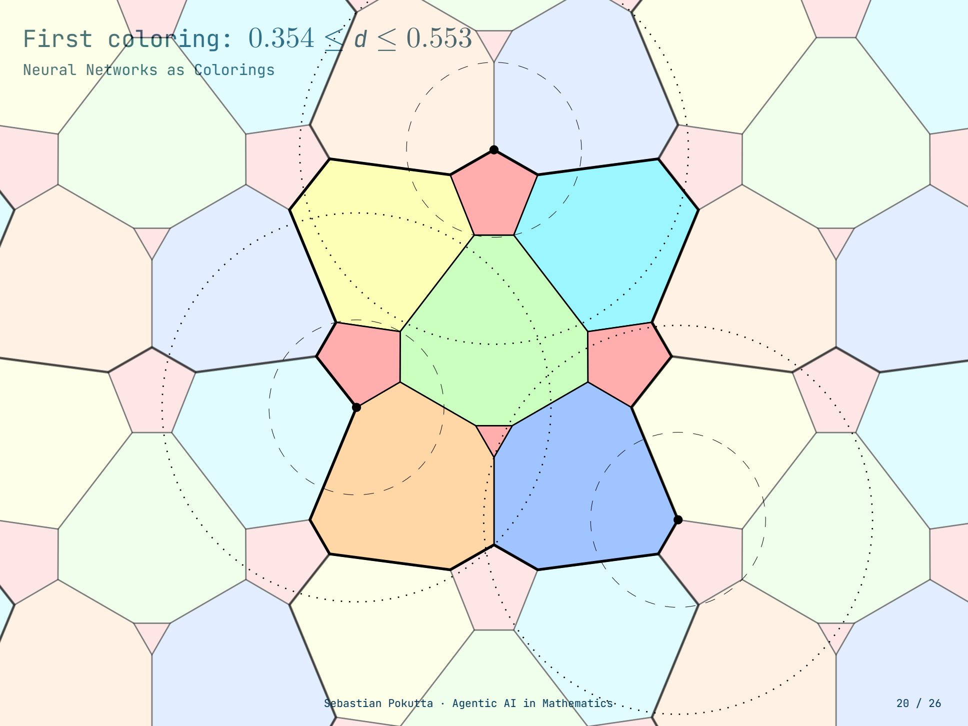
Deep Annealing approach provides two new colorings leading to...

Theorem

X_6 contains the closed interval [0.354, 0.657]. [Mundinger et al., 2024, 2025]

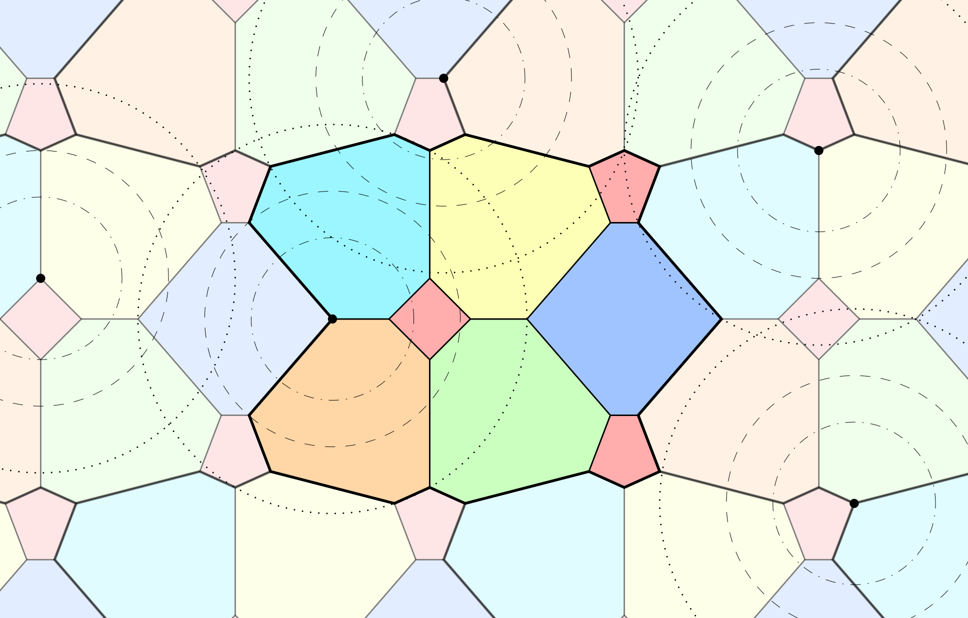
First coloring: $0.354 \leq d \leq 0.553$

Neural Networks as Colorings



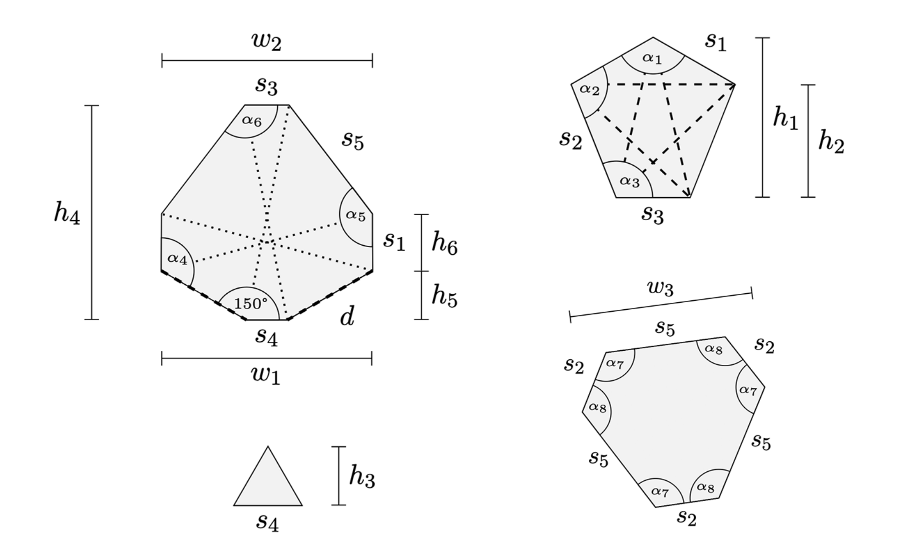
Second coloring: $0.418 \le d \le 0.657$

Neural Networks as Colorings



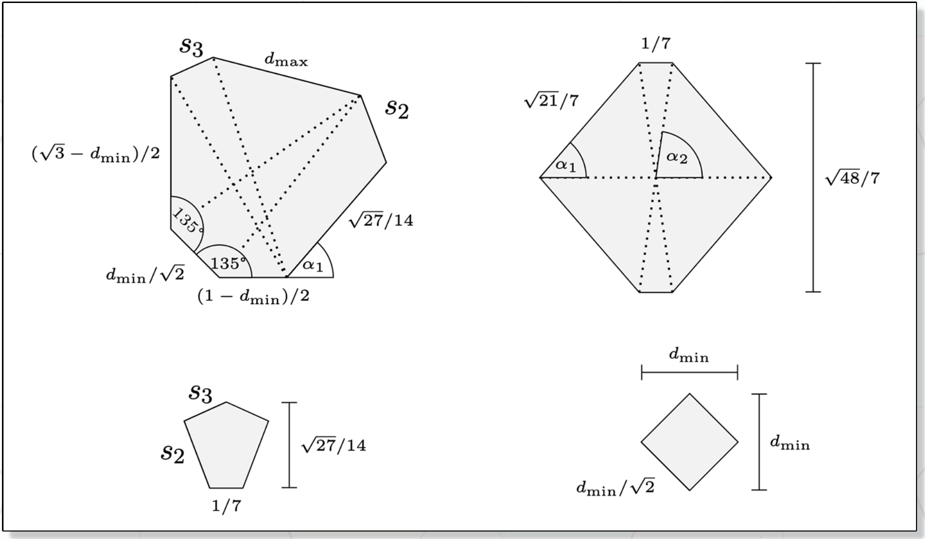
First coloring: exact components

Neural Networks as Colorings



Second coloring: exact components

Neural Networks as Colorings



Final Remarks

1. AI can be used in various ways in modern mathematical research workflows (actual discovery, verification, etc); beyond simple black-box prompting
2. Fully-automatic discovery of new mathematics might be possible in the future but relies on *strong* verification approaches
3. The agentic harness seems to be key: how does the agent receive feedback on its work, how is it guided, and which tools are available?
4. Empirically: the human-in-the-loop is crucial to guide the search

The promises of AI4MATH are great but need to go beyond simple black-box prompting “your favorite Erdős problem” (which then turns out having a solution that is already known)...

Thank you!

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